$F(T)$ gravity from higher dimensional theories and its cosmology

Main reference: **arXiv:1304.6191 [gr-qc]. To appear in Phys. Lett. B.**

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Contents

I. Introduction

Current cosmic accelerated expansion

- Recent observations of Type Ia Supernova (SNe Ia) has supported that the current expansion of the universe is accelerating.

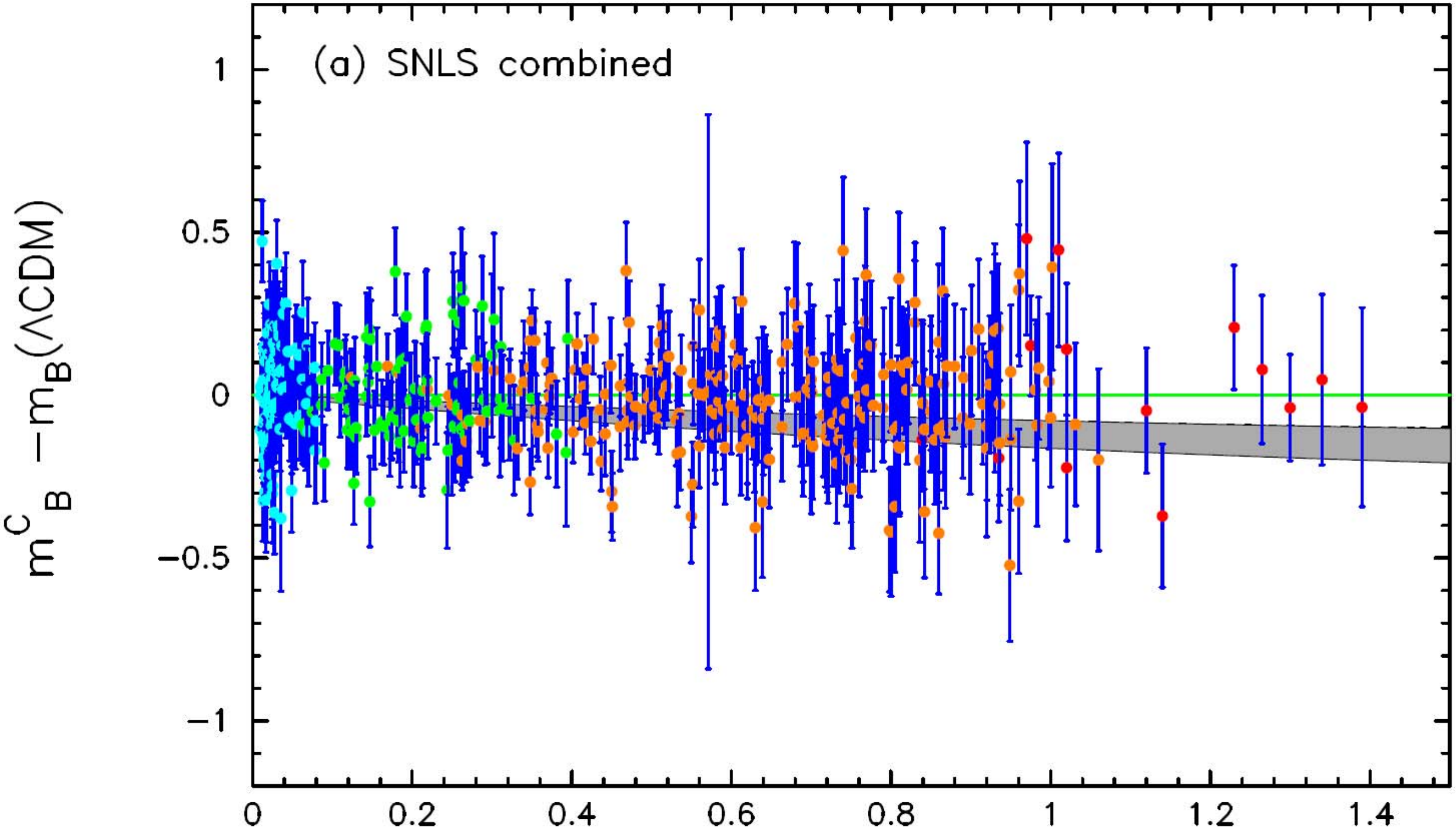
[Perlmutter *et al*. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999)]

[Riess *et al*. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998)]

2011 Nobel Prize in Physics

PLANCK 2013 results of SNLS

Magnitude residuals of the Λ CDM model that best fits the SNLS combined sample



$\leftarrow \Lambda$ **CDM model**

z : Redshift

From [Ade *et al*. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]].

- Suppose that the universe is strictly homogeneous and isotropic.

There are two approaches to explain the current accelerated expansion of the universe.

Reviews: E.g.,

[Copeland, Sami and Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006)]

[Nojiri and Odintsov, Phys. Rept. 505, 59 (2011); Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007)]

[Capozziello and Faraoni, *Beyond Einstein Gravity* (Springer, 2010)]

[Clifton, Ferreira, Padilla and Skordis, Phys. Rept. 513, 1 (2012)]

[KB, Capozziello, Nojiri and Odintsov, Astrophys. Space Sci. 342, 155 (2012)]

Gravitational field equation

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$\boxed{\textbf{Gravity}}$ $\boxed{\textbf{Matter}}$

$G_{\mu\nu}$: Einstein tensor

$T_{\mu\nu}$: Energy-momentum tensor

$$\kappa^2 \equiv 8\pi / {M_{\mathrm{Pl}}}^2$$

M_{Pl} : Planck mass

(1) **General relativistic approach** $\longrightarrow$ **Dark Energy**

(2) **Extension of gravitational theory**

(1) Candidates for dark energy

Cosmological constant, Scalar field, Fluid

(2) Extension of gravitational theory

- ***F*(*R*) gravity** $F(R)$: Arbitrary function of the Ricci scalar R

- **DGP braneworld scenario**

- **Galileon gravity**
- **Massive gravity**

- **Extended teleparallel gravity (*F*(*T*) gravity)**

$F(T)$: Arbitrary function of the torsion scalar T

Condition for accelerated expansion

Flat Friedmann-Lemaî tre-Robertson-Walker (FLRW) space-time

$$ds^2 = dt^2 - a^2(t) \sum_{i=1,2,3} \left(dx^i\right)^2$$

$a(>0)$: Scale factor

Equation of $a(t)$ for a single perfect fluid

ρ : Energy density

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6} \underline{(1+3w)\,\rho}$$

P : Pressure

$\dot{} = \partial/\partial t$

$\ddot{a} > 0$: **Accelerated expansion**

Cf. $w = -1$

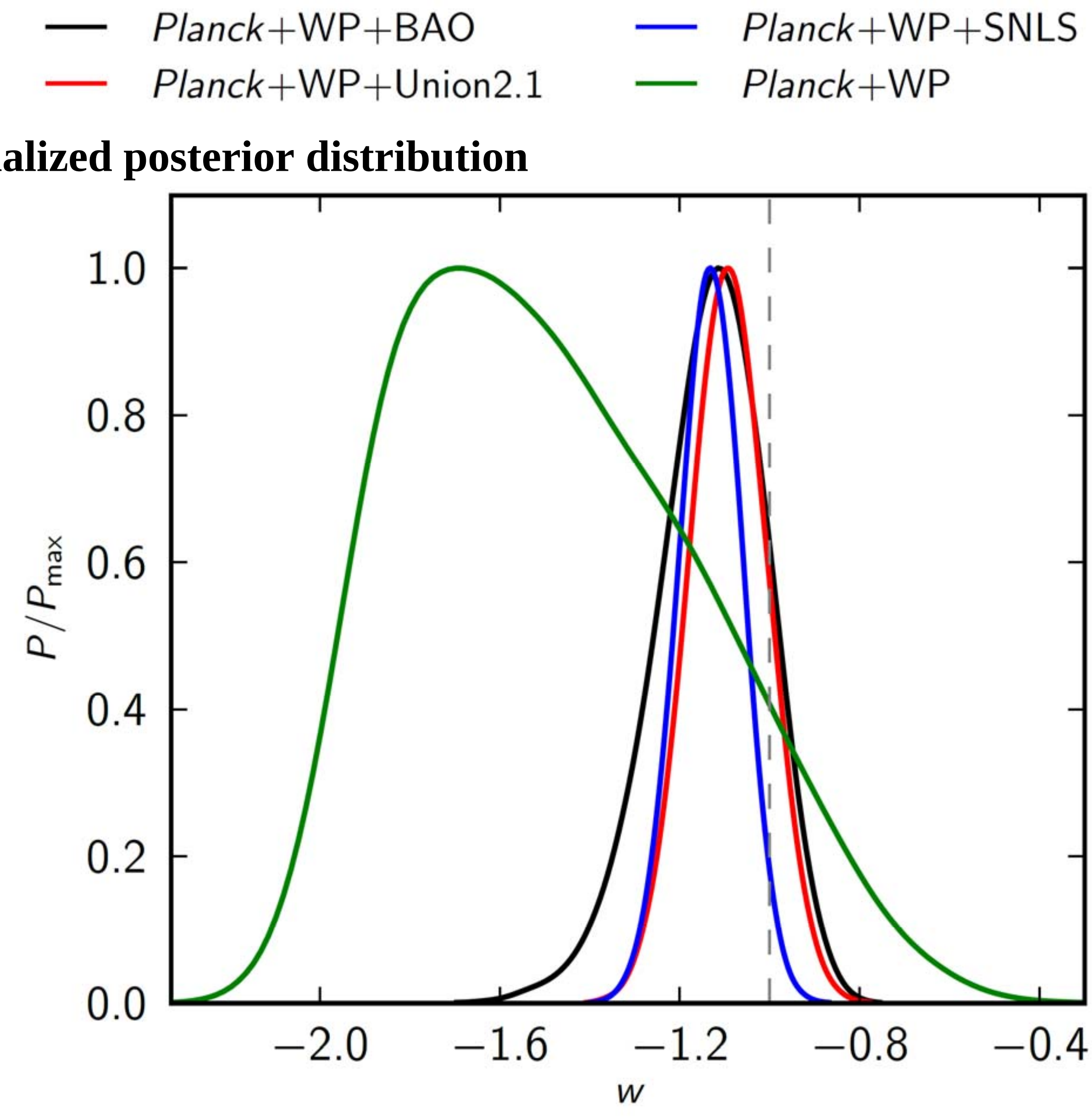
: Cosmological constant

$$\Rightarrow \boxed{w \equiv \frac{P}{\rho} < -\frac{1}{3}}$$

w : **Equation of state (EoS) parameter**

PLANCK data for the current w

Marginalized posterior distribution



From [Ade *et al*. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]].

$$w = \text{constant}$$

WP: WMAP

BAO: Baryon Acoustic Oscillation

$$w = -1.13^{+0.24}_{-0.25} \quad (95\%;\ Planck\text{+WP+BAO})$$

$$w = -1.09 \pm 0.17 \quad (95\%;\ Planck\text{+WP+Union2.1})$$

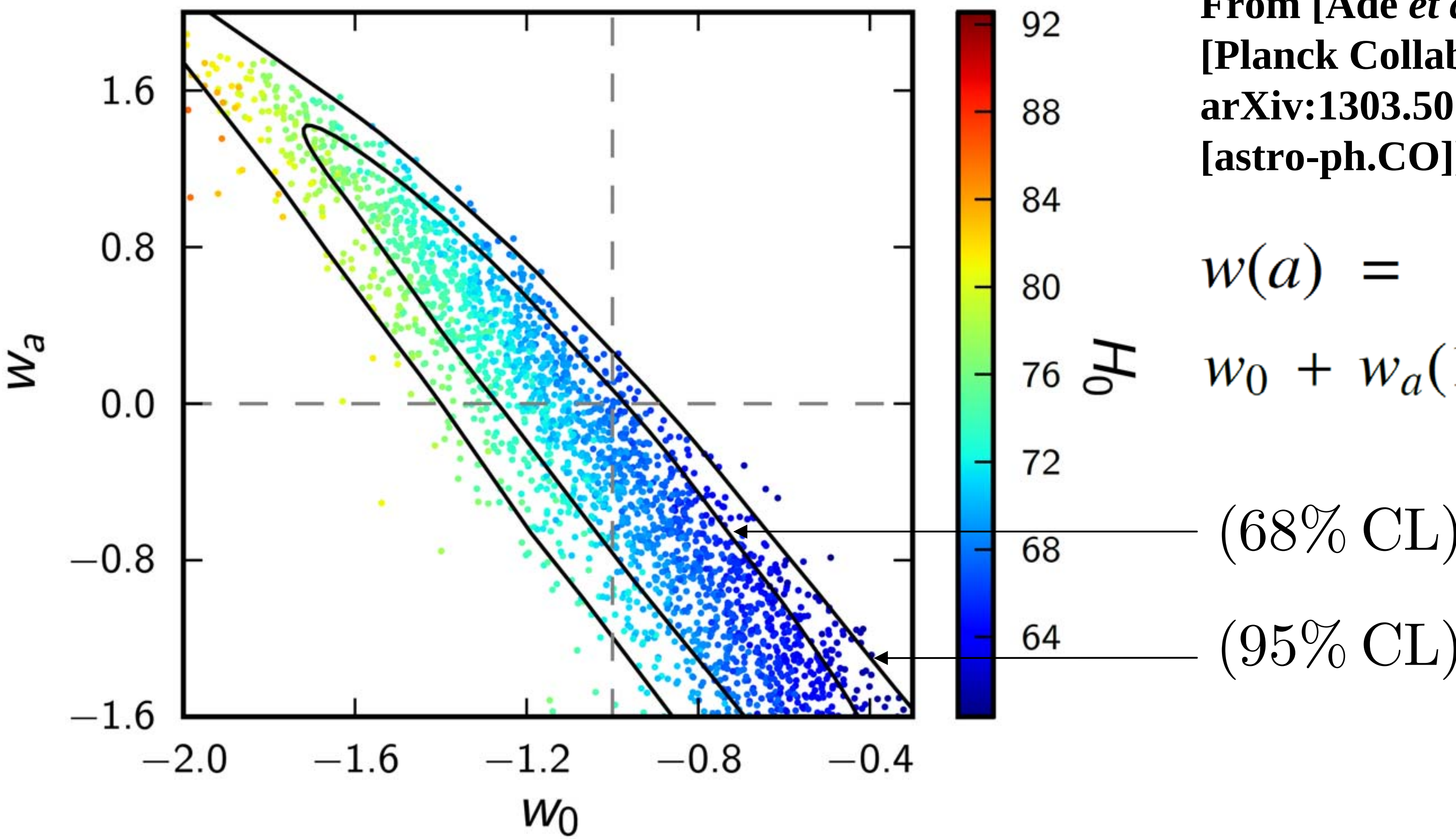
$$\underline{w = -1.13^{+0.13}_{-0.14}} \quad (95\%;\ Planck\text{+WP+SNLS})$$

$$\underline{w = -1.24^{+0.18}_{-0.19}} \quad (95\%;\ Planck\text{+WP+}H_0)$$

* Hubble constant (H_0) measurement

PLANCK data for the time-dependent w

2D Marginalized posterior distribution



From [Ade *et al.* [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]].

$$w(a) = w_0 + w_a(1-a)$$

$$w_0 = -1.04^{+0.72}_{-0.69} \quad (95\%;\ Planck\text{+WP+BAO})$$

$$w_a < 1.32 \quad (95\%;\ Planck\text{+WP+BAO})$$

Motivation and Subject

It is meaningful to investigate theoretical features of modified gravity theories.

→ **We concentrate on cosmological aspects of $F(T)$ gravity.**

- **We explore the four-dimensional effective $F(T)$ gravity originating from higher-dimensional space-time theories, in particular the Kaluza-Klein (KK) and Randall-Sundrum (RS) theories.**

II. $F(T)$ gravity

Teleparallelism

- $g_{\mu\nu} = \eta_{AB} e_\mu^A e_\nu^B$

η_{AB} : Minkowski metric

$e_A(x^\mu)$: Orthonormal tetrad components

- $T^\rho_{\mu\nu} \equiv \Gamma^{\rho(W)}_{\mu\nu} - \Gamma^{\rho(W)}_{\nu\mu} = e^\rho_A \left(\partial_\mu e^A_\nu - \partial_\nu e^A_\mu\right)$: **Torsion tensor**

$$\Gamma^{\rho(W)}_{\mu\nu} \equiv e^\rho_A \partial_\mu e^A_\nu$$: Weitzenböck connection

* μ and ν are coordinate indices on the manifold and also run over 0, 1, 2, 3, and $e_A(x^\mu)$ forms the tangent vector of the manifold.

* An index A runs over 0, 1, 2, 3 for the tangent space at each point x^μ of the manifold.

$$\Rightarrow \quad T \equiv S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}$$: **Torsion scalar**

$$K^{\mu\nu}{}_\rho \equiv -\frac{1}{2}\left(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}\right)$$: Contorsion tensor

$$S_\rho{}^{\mu\nu} \equiv \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho\, T^{\alpha\nu}{}_\alpha - \delta^\nu_\rho\, T^{\alpha\mu}{}_\alpha\right)$$

[Hehl, Von Der Heyde, Kerlick and Nester, Rev. Mod. Phys. 48, 393 (1976)]

[Hayashi and Shirafuji, Phys. Rev. D 19, 3529 (1979)
[Addendum-ibid. D 24, 3312 (1981)]]

Extended teleparallel gravity

Action

$$S = \int d^4x|e| \left(\underline{\frac{F(T)}{2\kappa^2}} + \mathcal{L}_{\mathrm{M}} \right)$$: ***F*(*T*) gravity**

$$|e| = \det\left(e^A_\mu\right) = \sqrt{-g}$$

$\mathcal{L}_{\mathrm{M}}$: Matter Lagrangian

${T^{(\mathrm{M})}}_\rho{}^\nu$: Energy-momentum tensor of matter

Gravitational field equation

$$\frac{1}{e}\partial_\mu\left(e {S_A}^{\mu\nu}\right)F' - e^\lambda_A {T^\rho}_{\mu\lambda} {S_\rho}^{\nu\mu} F' + {S_A}^{\mu\nu} \partial_\mu (T) F'' + \frac{1}{4} e^\nu_A F = \frac{\kappa^2}{2} e^\rho_A {T^{(\mathrm{M})}}_\rho{}^\nu$$

∗ A prime denotes a derivative with respect to T.

[Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]

- **Gravitational field equation in *F*(*T*) gravity is 2nd order, while it is 4th order in *F*(*R*) gravity.**

- For the flat FLRW space-time with the metric:

$$ds^2 = dt^2 - a^2(t) \sum_{i=1,2,3} \left(dx^i\right)^2 \Longrightarrow \underline{T = -6H^2}$$

$$g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$$

$$H \equiv \frac{\dot{a}}{a}$$

$$e_\mu^A = (1, a, a, a)$$

: Hubble parameter

Gravitational field equations

ρ_{DE} : Dark energy density

P_{DE} : Pressure of dark energy

$\rho_{\mathrm{M}},\ P_{\mathrm{M}}$

: Energy density and pressure of dark energy

$$H^2 = \frac{\kappa^2}{3}\left(\rho_{\mathrm{M}} + \rho_{\mathrm{DE}}\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\rho_{\mathrm{M}} + P_{\mathrm{M}} + \rho_{\mathrm{DE}} + P_{\mathrm{DE}}\right)$$

$$\rho_{\mathrm{DE}} = \frac{1}{2\kappa^2}\left(-T - F + 2TF'\right)$$

$$P_{\mathrm{DE}} = -\frac{1}{2\kappa^2}\left[4\left(1 - F' - 2TF''\right)\dot{H} - T - F + 2TF'\right]$$

Continuity equation

$$\dot{\rho}_{\mathrm{DE}} + 3H\left(\rho_{\mathrm{DE}} + P_{\mathrm{DE}}\right) = 0$$

Example of $F(T)$ gravity model

$$F(T) = T + \gamma \left[T_0 \left(\frac{uT_0}{T} \right)^{-1/2} \ln \left(\frac{uT_0}{T} \right) - T \left(1 - e^{uT_0/T} \right) \right]$$

$$\gamma \equiv \frac{1 - \Omega_{\mathrm{m}}^{(0)}}{2u^{-1/2} + \left[1 - (1 - 2u)\, e^{u} \right]}$$

$u (> 0)$: Positive constant

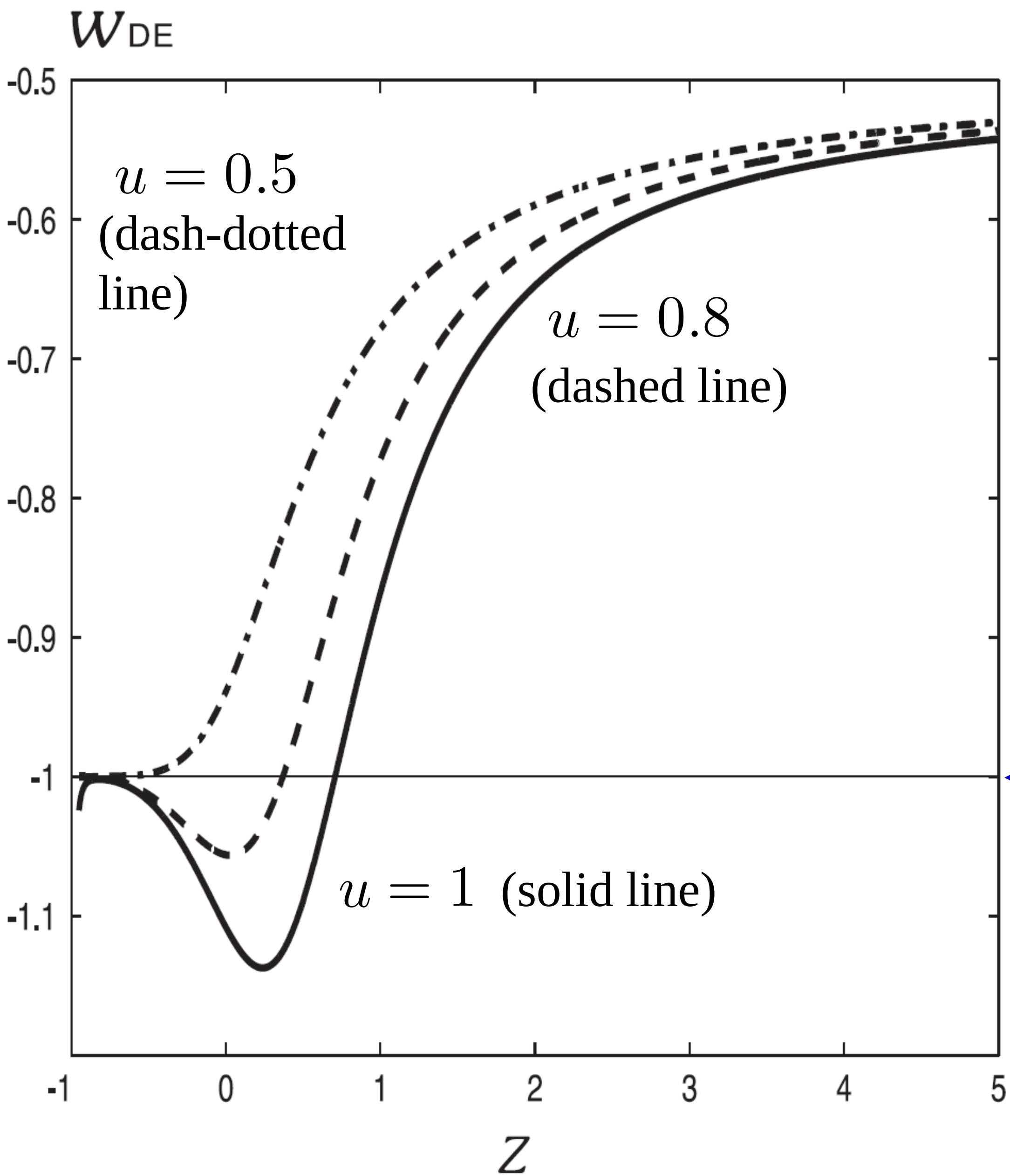
$$\Omega_{\mathrm{m}}^{(0)} \equiv \rho_{\mathrm{m}}^{(0)} / \rho_{\mathrm{crit}}^{(0)} , \qquad T_0 = T(z = 0)$$

$$\rho_{\mathrm{crit}}^{(0)} = 3H_0^2 / \kappa^2$$

- **The model contains only one parameter u if one has the value of $\Omega_{\mathrm{m}}^{(0)}$.**

[KB, Geng, Lee and Luo, JCAP 1101, 021 (2011)]

Cosmological evolutions of $w_{\rm DE}$

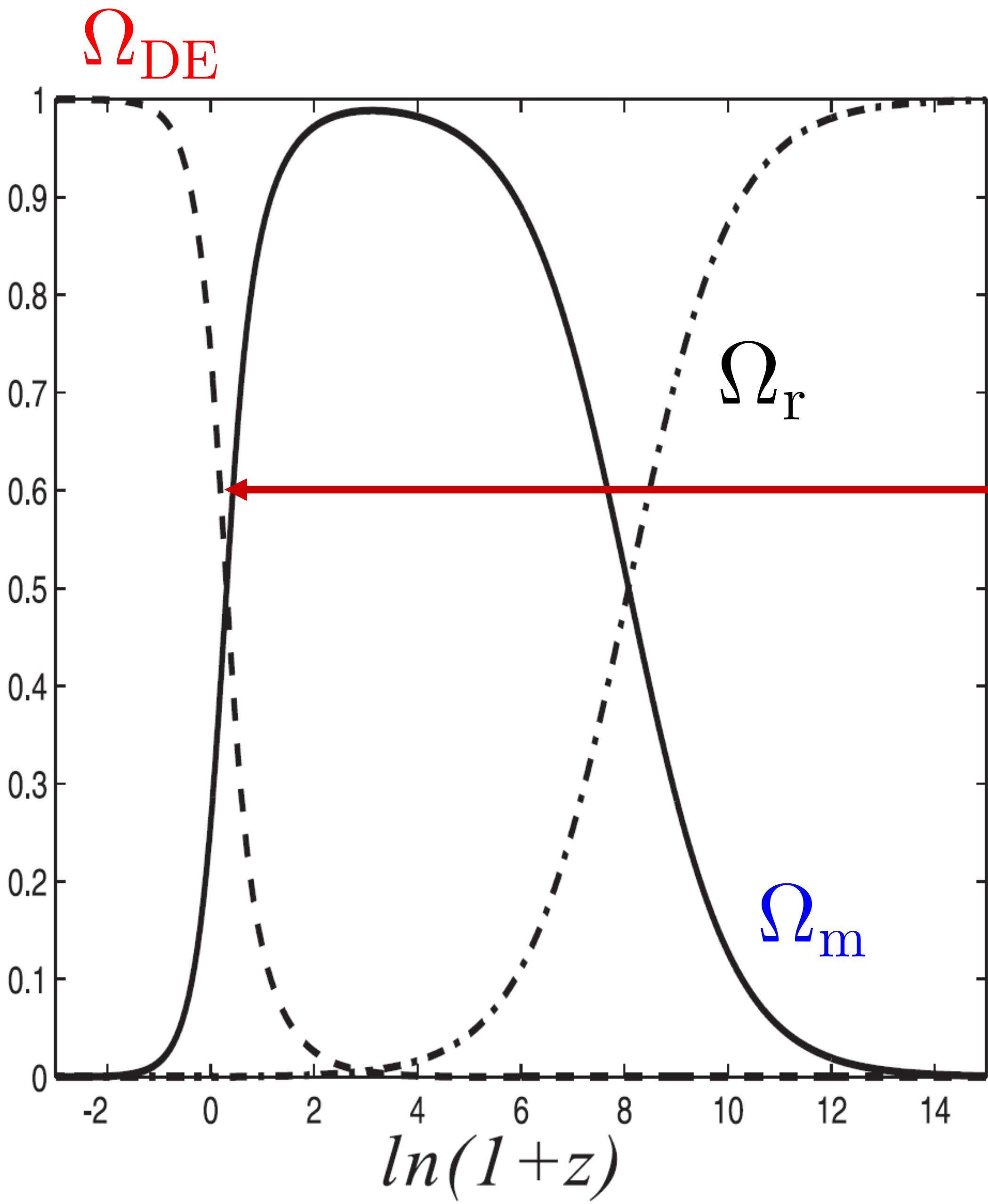


From [KB, Geng, Lee and Luo, JCAP 1101, 021 (2011)].

$w_{\rm DE} = -1$

Crossing of the phantom divide

Cosmological evolutions of $\Omega_{\rm DE}$, $\Omega_{\rm m}$ and $\Omega_{\rm r}$



$u = 1$

Dark energy dominated stage

From [KB, Geng, Lee and Luo, JCAP 1101, 021 (2011)].

III. From Kaluza-Klein (KK) theory

Action in five-dimensional space-time

$$ {}^{(5)}S = \int d^5x \left| {}^{(5)}e \right| \frac{F({}^{(5)}T)}{2\kappa_5^2} $$

$$ {}^{(5)}T \equiv \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{cba} - T_{ab}{}^{a} T^{cb}{}_{c} $$

$$ {}^{(5)}e = \sqrt{{}^{(5)}g} $$

* $a, b, \ldots$ run over $0, 1, 2, 3, 5$.

$$ \kappa_5^2 \equiv 8\pi G_5 = \left({}^{(5)}M_{\mathrm{Pl}}\right)^{-3} $$

* "5" denotes the component of the fifth coordinate.

- **The superscript or subscript of (5) or 5 mean the quantities in the five-dimensional space-time.**

[Capozziello, Gonzalez, Saridakis and Vasquez, JHEP <u>1302</u>, 039 (2013)]

Original KK compactification scenario

- One of the dimensions of space is compactified to a small circle and the four-dimensional space-time is extended infinitely.
- The radius of the fifth dimension is taken to be of order of the Planck length in order for the KK effects not to be seen.

⇨ **The size of the circle is so small that phenomena in sufficiently low energies cannot be detected.**

[Appelquist, Chodos and Freund, *Modern Kaluza-Klein Theories* (Addison-Wesley, Reading, 1987)]

[Fujii and Maeda, The Scalar-Tensor Theory of Gravitation (Cambridge University Press, Cambridge, United Kingdom, 2003)]

Metric in five-dimensional space-time

$$ {}^{(5)}g_{ab} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -\phi^2 \end{pmatrix}, $$

$\phi \equiv \varphi/\varphi_*$: Dimensionless homogeneous scalar field

φ_* : Fiducial value of φ

$$ \phi^2 = \mathcal{R}^2\theta^2 $$

$\mathcal{R}$: Rradius of the compactified space

θ : Dimensionless coordinates such as an angle

$$ \sqrt{{}^{(5)}g} = \sqrt{-g}\mathcal{R}\sqrt{\hat{g}} $$

$\hat{g}$: Determinant of the metric corresponding to the pure geometrical part represented by θ

$V_{\rm com} = \int \hat{g} d\theta$: Compactified space volume

Effective action in the four-dimensional space-time

$$e_a^A = \mathrm{diag}(1,1,1,1,\phi)$$

$$\eta_{ab} = \mathrm{diag}(1,-1,-1,-1,-1)$$

$$\Rightarrow \quad S_{\mathrm{KK}}^{\mathrm{eff}} = \int d^4x|e|\frac{1}{2\kappa^2}\phi F(T+\phi^{-2}\partial_\mu\phi\partial^\mu\phi)$$

$$\left|{}^{(5)}e\right| = \phi|e|$$

- **Our KK reduced action is compatible with the results in the following reference:**

[Fiorini, Gonzalez and Vasquez, arXiv:1304.1912 [gr-qc]].

Case of teleparallelism with a positive cosmological constant

- $F(T) = T - 2\Lambda_4$, $\Lambda_4 (> \ 0)$: Cosmological constant

- We define σ as $\phi \equiv \xi \sigma^2$, $\xi = 1/4$

$$\Rightarrow \quad S_{\mathrm{KK}}^{\mathrm{eff}}|_{F(T)=T-2\Lambda_4} =$$

$$\int d^4x |e| \left(1/\kappa^2\right) \left[(1/8)\, \sigma^2 T + \underline{(1/2)\, \partial_\mu \sigma \partial^\mu \sigma} - \Lambda_4\right]$$

Canonical kinetic term

Cosmology in the flat FLRW space-time

Gravitational field equations

$$(1/2)\,\dot{\sigma}^2 \;-\; (3/4)\,H^2\sigma^2 \;+\; \Lambda_4 \;=\; 0$$

$$\dot{\sigma}^2 \;+\; H\sigma\dot{\sigma} \;+\; (1/2)\,\dot{H}\sigma^2 \;=\; 0$$

$$\Longrightarrow \quad (3/2)\,H^2\sigma^2 - 2\Lambda_4 + H\sigma\dot{\sigma} + (1/2)\,\dot{H}\sigma^2 = 0$$

Equation of motion of σ

$$\ddot{\sigma} + 3H\dot{\sigma} + (3/2)\,H^2\sigma = 0$$

Cf. [Geng, Lee, Saridakis and Wu, Phys. Lett. B 704, 384 (2011)]

Solution

$$H = H_{\rm inf} = \text{constant}(> 0)$$

$$\sigma = b_1 \left(t/t_1\right) + b_2 \qquad b_1, \quad b_2(> 0), \quad t_1 \quad \text{: Constants}$$

- In the limit $t \longrightarrow 0$, we can find approximate expressions

$$H_{\rm inf} \approx \left(2/b_2\right) \sqrt{\Lambda_4/3}$$

$$\sigma \approx b_2$$

$$b_1 \approx -\left(1/2\right) b_2 H_{\rm inf} t_1 \approx -\sqrt{\Lambda_4/3} t_1$$

$$a \approx \bar{a} \exp\left(H_{\rm inf} t\right), \qquad \bar{a}(> 0)$$

⇨ **An exponential inflation can be realized.**

IV. From the Randall-Sundrum (RS) theory

The RS type-I and II models

- In the RS type-I model, there are a positive tension brane at $y = 0$ and a negative one at $y = s$, where y is the fifth direction.

$$d\tilde{s}^2 = \mathrm{e}^{-2|y|/l} g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2, \qquad l = \sqrt{-6/\Lambda_5}$$

$\mathrm{e}^{-2|y|/l}$: Warp factor

$\Lambda_5 (< 0)$: Negative cosmological constant in the bulk

$$\Big\Downarrow \quad s \to \infty$$

- In the RS type-II model, there is only one brane with the positive tension floating in the AdS bulk space.

[Randall and Sundrum, Phys. Rev. Lett. 83, 3370 (1999); 4690 (1999)]

Cf. [Garriga and Tanaka, Phys. Rev. Lett. 84, 2778 (2000)]

Settings in the RS type-II model

- We start with the equation in the five-dimensional space-time with the brane whose tension is a positive constant.

- We consider that the vacuum solution in the five-dimensional space-time is the AdS one, and that the brane configuration is consistent with the equation in the five-dimensional space-time.

$\Rightarrow$ This implies that the brane configuration with a positive constant tension connecting two vacuum solutions in the five-dimensional space-time, namely, the condition of the configuration is nothing but the equation for the brane.

Procedures in the RS type-II model

Pioneering work:

[Shiromizu, Maeda and Sasaki, Phys. Rev. D 62, 024012 (2000)]

⇨ Application to teleparallelism:

[Nozari, Behboodi and Akhshabi, Phys. Lett. B 723, 201 (2013)]

(i) The corresponding Gauss-Codazzi equations in teleparallelism, namely, the induced equations on the brane, is examined by using the projection vierbein of the five-dimensional space-time quantities into the four-dimensional space-time brane.

(ii) The Israel's junction conditions to connect the left-side and right-side bulk spaces sandwiching the brane are investigated.

- The first junction condition is that the vierbeins induced on the brane from the left side and right side of the brane should be the same with each other.

- Moreover, the second junction condition is that the difference of the tensor $S_{\rho}{}^{\mu\nu}$ between the left side and right side of the brane comes from the energy-momentum tensor of matter, which is confined in the brane.

(iii) Provided that there exists Z_2 symmetry, i.e., $y \leftrightarrow -y$, in the five-dimensional space-time, the quantities on the left and right sides of the brane are explored.

- The difference between the scalar curvature and the torsion scalar is a total derivative of the torsion tensor.

  $\longrightarrow$ This may affect the boundary.

- It has been shown that in comparison with the gravitational field equations in general relativity, the induced gravitational field equations on the brane have new terms, which comes from the additional degrees of freedom in teleparallelism.

- These extra terms correspond to the projection on the brane of the vector portion of the torsion tensor in the bulk.

Cosmology in the flat FLRW space-time

Friedmann equation on the brane

$$H^2 \frac{dF(T)}{dT} = -\frac{1}{12}\left[F(T) \underline{-4\Lambda} - 2\kappa^2 \rho_{\mathrm{M}} \underline{- \left(\frac{\kappa_5^2}{2}\right)^2 \mathcal{Q} \rho_{\mathrm{M}}^2}\right]$$

$$\mathcal{Q} \equiv \left(11 - 60 w_{\mathrm{M}} + 93 w_{\mathrm{M}}^2\right)/4 \longleftarrow$$

includes the contributions from teleparallelism, which do not exist in general relativity.

$$w_{\mathrm{M}} \equiv P_{\mathrm{M}}/\rho_{\mathrm{M}}$$

$\Lambda \equiv \Lambda_5 + \left(\kappa_5^2/2\right)^2 \lambda^2$: Eeffective cosmological constant in the brane

$\lambda(> 0)$: the tension of the brane

$$G = [1/(3\pi)] \left(\kappa_5^2/2\right)^2 \lambda$$

Case (1)

$$F(T) = T - 2\Lambda_5$$

* At the dark energy completely dominated stage, we can consider $w_{\mathrm{M}} = 0$.

$$\longrightarrow \ H = H_{\mathrm{DE}} = \sqrt{\Lambda_5 + \kappa_5^4 \lambda^2 / 6} = \mathrm{constant}$$

$$a(t) = a_{\mathrm{DE}} \exp\left(H_{\mathrm{DE}} t\right), \qquad a_{\mathrm{DE}} (> 0)$$

$\Rightarrow$ **An approximate de Sitter solution on the brane can be realized.**

Cf. Other solution

For $F(T) = T$, $\Lambda = 0$, $\mathcal{Q} = 8/3$, and $w_{\mathrm{M}} = -5.5 \times 10^{-3}$,

$$H^2 = \left(\kappa^2/3\right) \rho_{\mathrm{M}} \left[1 + \rho_{\mathrm{M}} / \left(2\lambda\right)\right]$$

[Astashenok, Elizalde, de Haro, Odintsov and Yurov, arXiv:1301.6344 [gr-qc]]

Case (2)

$\bar{M}$: Mass scale

α : Constant

$$F(T) = T^2/\bar{M}^2 + \alpha\Lambda_5$$

$$\longrightarrow \quad H = H_{\mathrm{DE}} = \left[\left(\bar{M}^2/108\right)\mathcal{J}\right]^{1/4} = \mathrm{constant}$$

$$\mathcal{J} \equiv (\alpha - 4)\Lambda_5 - 4\left(\kappa_5^2/2\right)^2 \lambda^2$$

$$a(t) = a_{\mathrm{DE}} \exp\left(H_{\mathrm{DE}} t\right), \qquad a_{\mathrm{DE}} (> 0)$$

$$\mathcal{J} (\geq 0) \Longrightarrow \alpha \geq 4 + \left(\kappa_5^2 \lambda^2\right)/\Lambda_5$$

⇨ **Similar approximate de Sitter solution on the brane can be obtained.**

V. Summary

- Four-dimensional effective $F(T)$ gravity coming from the five-dimensional KK and RS space-time theories have been studied.

- **With the KK reduction, the four-dimensional effective theory of $F(T)$ gravity coupling to a scalar field has been built.**

- **For the RS type-II model, the contribution of $F(T)$ gravity appears on the four-dimensional FLRW brane.**

- **Inflation and the dark energy dominated stage can be realized in the KK and RS theories, respectively, due to the effect of only the torsion in teleparallelism without that of the curvature.**

Backup Slides

General relativistic approach

(i) Cosmological constant

(ii) Scalar field :

- **x-matter, Quintessence** ← **Canonical field**

[Chiba, Sugiyama and Nakamura, Mon. Not. Roy. Astron. Soc. 289, L5 (1997)]
[Caldwell, Dave and Steinhardt, Phys. Rev. Lett. 80, 1582 (1998)]
Cf. Pioneering work: [Fujii, Phys. Rev. D 26, 2580 (1982)]

- **Phantom** ← **Wrong sign kinetic term**

[Caldwell, Phys. Lett. B 545, 23 (2002)]

- **K-essence** ← **Non canonical kinetic term**

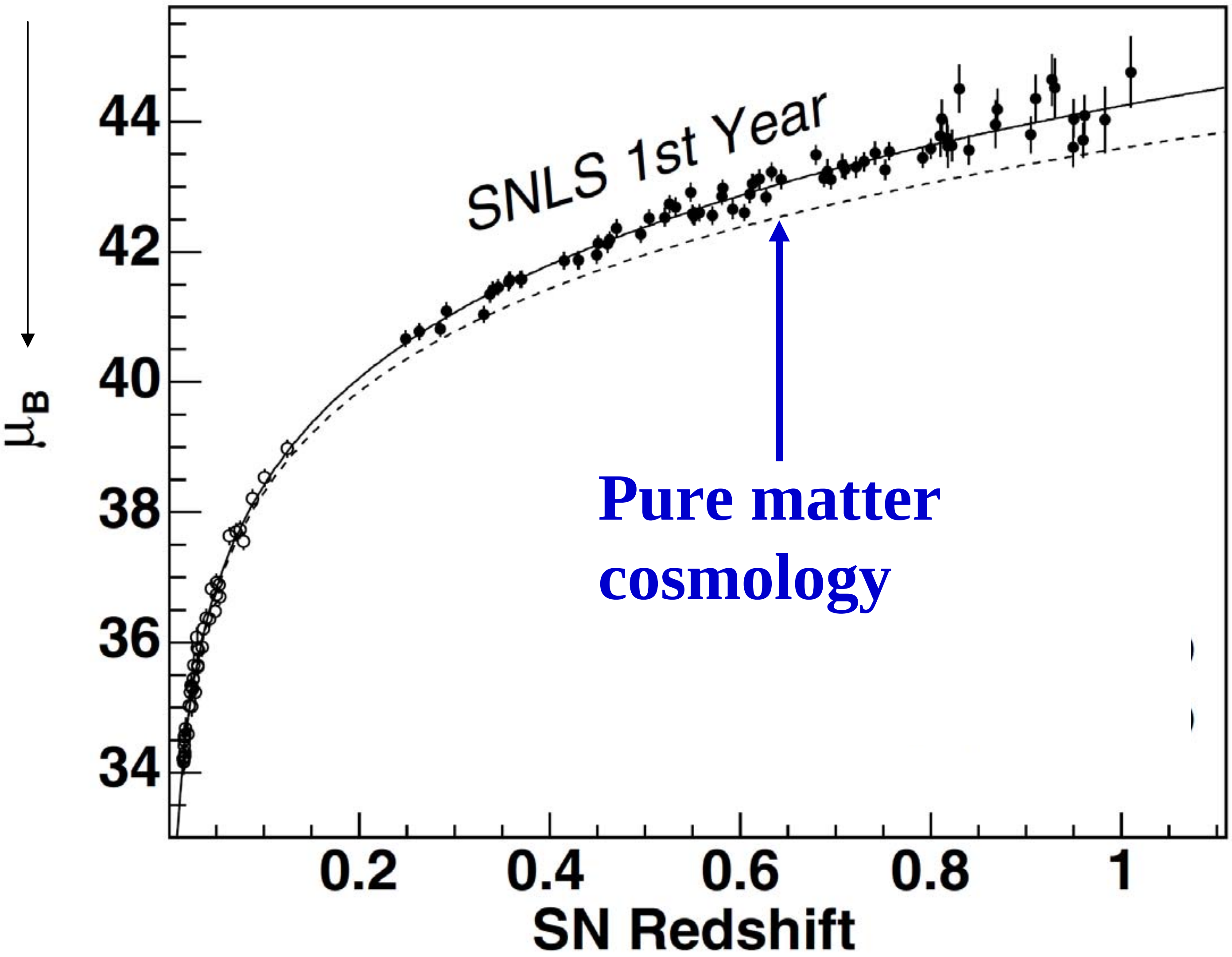
[Chiba, Okabe and Yamaguchi, Phys. Rev. D 62, 023511 (2000)]
[Armendariz-Picon, Mukhanov and Steinhardt, Phys. Rev. Lett. 85, 4438 (2000)]

- **Tachyon** ← **String theories** ∗ **The mass squared is negative.**

[Padmanabhan, Phys. Rev. D 66, 021301 (2002)]

SNLS data

Distance estimator

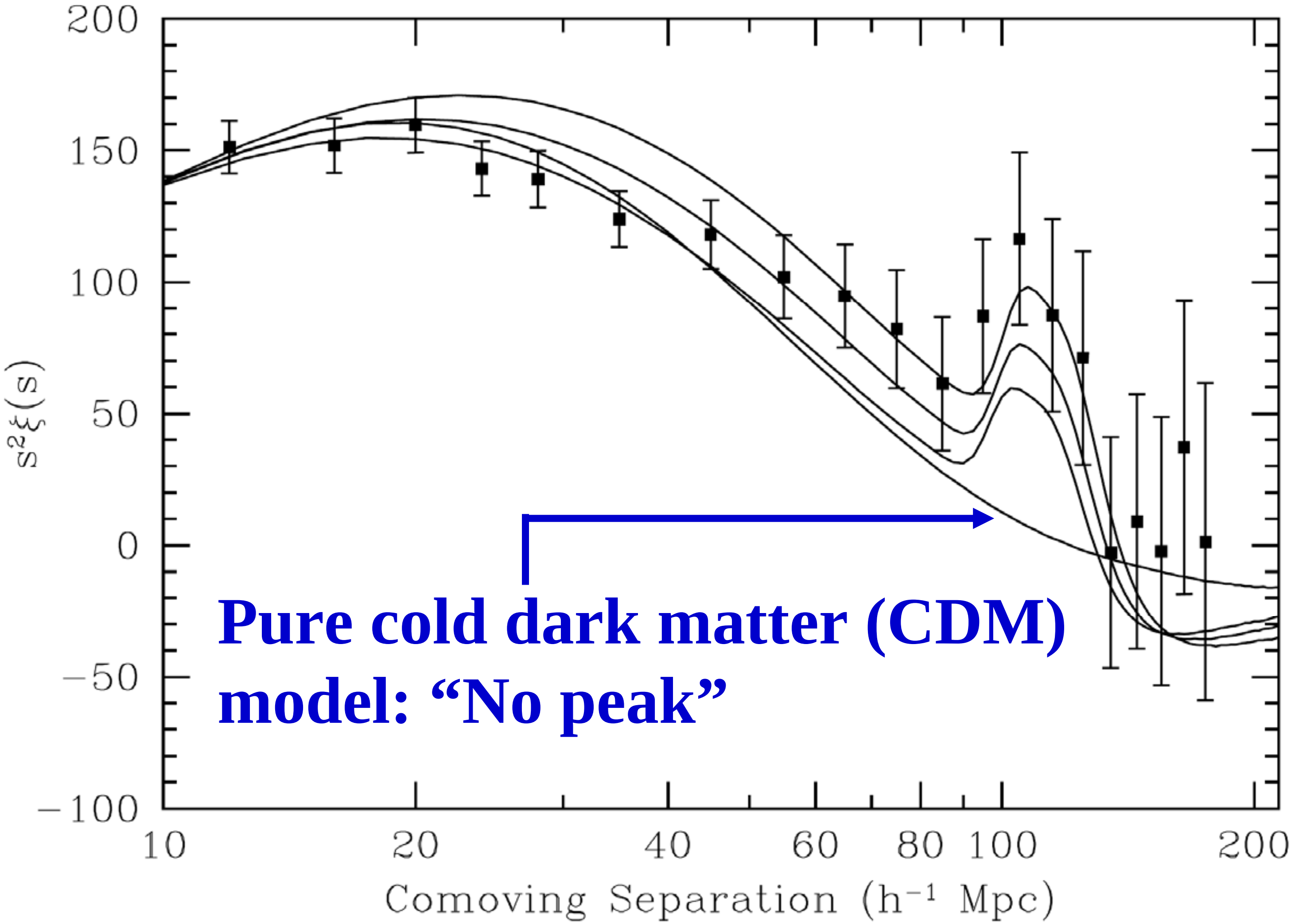


Flat Λ cosmology

z **: Redshift**

From [Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)].

Baryon acoustic oscillation (BAO)



Special pattern in the large-scale correlation function of Sloan Digital Sky Survey (SDSS) luminous red galaxies

From [Eisenstein *et al*. [SDSS Collaboration], Astrophys. J. 633, 560 (2005)].

Cf. [Yamamoto, astro-ph/0110596; Astrophys. J. 595, 577 (2003)]
[Matsubara and Szalay, Phys. Rev. Lett. 90, 021302 (2003)]

9-year WMAP data of current w

[Hinshaw *et al.*, arXiv:1212.5226 [astro-ph.CO]]

For constant w :

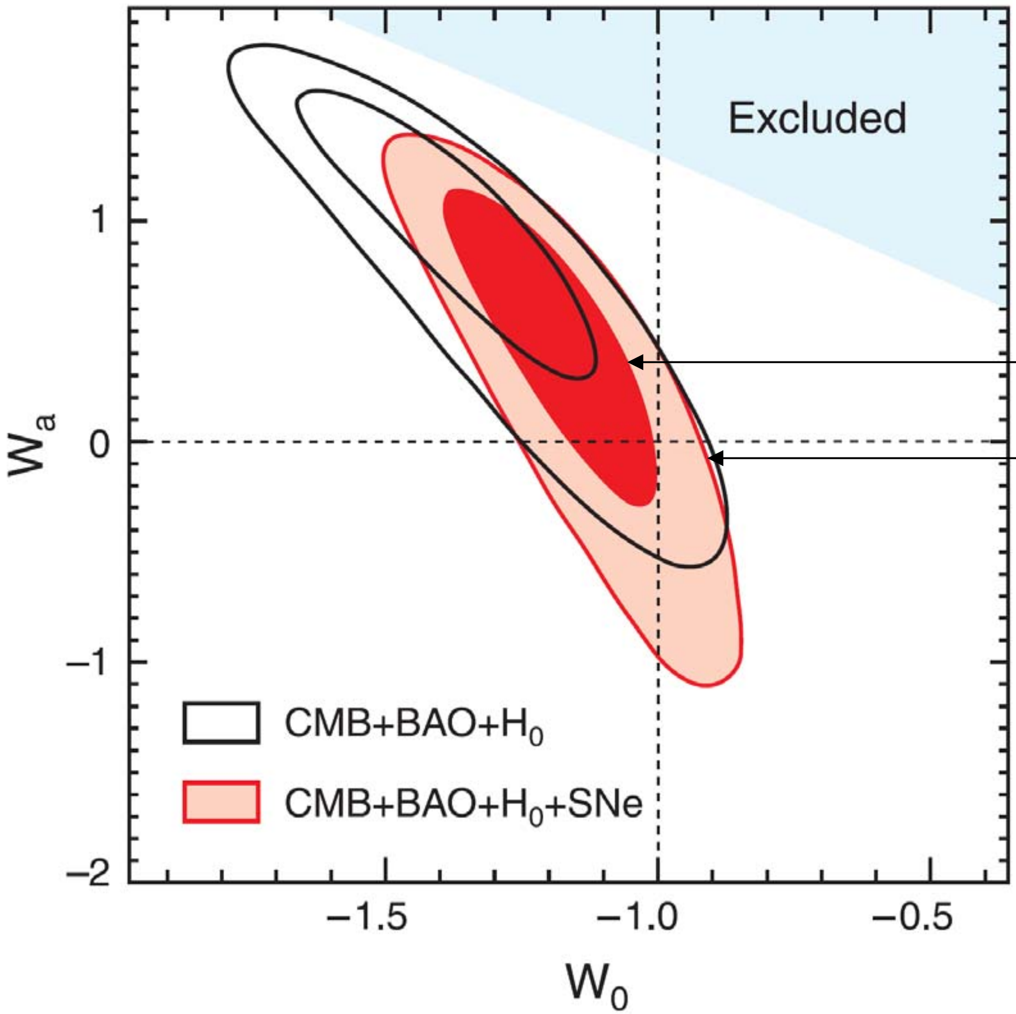
$$w = \begin{cases} -1.084 \pm 0.063 & \text{(flat)} \\ -1.122^{+0.068}_{-0.067} & \text{(non-flat)} \end{cases} \quad (68\%\ \text{CL})$$

(From $WMAP$+eCMB+BAO+H_0+SNe.)

* Hubble constant (H_0) measurement

Time-dependent w

$$w(a) = w_0 + w_a(1 - a)$$



(68% CL)

(95% CL)

From [Hinshaw *et al.*, arXiv:1212.5226 [astro-ph.CO]].

w_0: Current value of w

(From $WMAP$+eCMB +BAO+H_0+SNe.)

For the flat universe:

$$w_0 = -1.17^{+0.13}_{-0.12}, \quad w_a = 0.35^{+0.50}_{-0.49} \quad (68\% \text{ CL})$$

(iii) Fluid :

- **(Generalized) Chaplygin gas**

Equation of state (EoS): $P = -A/\rho^u$

$A > 0, \ u$: Constants

ρ : Energy density

P : Pressure

[Kamenshchik, Moschella and Pasquier, Phys. Lett. B 511, 265 (2001)] $\longleftarrow (u = 1)$

[Bento, Bertolami and Sen, Phys. Rev. D 66, 043507 (2002)]

Extension of gravitational theory

- ***F*(*R*) gravity** $\longleftarrow$ $F(R)$: Arbitrary function of the Ricci scalar R

Cf. Application to inflation: **[Starobinsky, Phys. Lett. B 91, 99 (1980)]**

[Capozziello, Cardone, Carloni and Troisi, Int. J. Mod. Phys. D 12, 1969 (2003)]

[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]

[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]

- **Scalar-tensor theories** $\longleftarrow$ $f_1(\phi)R$

$f_i(\phi)$ $(i=1,2)$: Arbitrary function of a scalar field ϕ

[Boisseau, Esposito-Farese, Polarski and Starobinsky, Phys. Rev. Lett. 85, 2236 (2000)]

[Gannouji, Polarski, Ranquet and Starobinsky, JCAP 0609, 016 (2006)]

- **Ghost condensates scenario**

[Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405, 074 (2004)]

- **Higher-order curvature term**

Gauss-Bonnet invariant with a coupling to a scalar field: $f_2(\phi)\mathcal{G}$

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

: Gauss-Bonnet invariant

$R_{\mu\nu}$: Ricci curvature tensor

$R_{\mu\nu\rho\sigma}$: Riemann tensor

[Nojiri, Odintsov and Sasaki, Phys. Rev. D 71, 123509 (2005)]

- $f(\mathcal{G})$ **gravity** $\longleftarrow \dfrac{R}{2\kappa^2} + f(\mathcal{G})$

$$\kappa^2 \equiv 8\pi G$$

G : Gravitational constant

[Nojiri and Odintsov, Phys. Lett. B 631, 1 (2005)]

· DGP braneworld scenario

[Dvali, Gabadadze and Porrati, Phys. Lett B 485, 208 (2000)]

[Deffayet, Dvali and Gabadadze, Phys. Rev. D 65, 044023 (2002)]

· Non-local gravity $\longleftarrow$ $\frac{1}{2\kappa^2} f\left(\Box^{-1} R\right)$ **: Quantum effects**

[Deser and Woodard, Phys. Rev. Lett. 99, 111301 (2007)]

$\Box$: Covariant d'Alembertian

[Nojiri and Odintsov, Phys. Lett. B 659, 821 (2008)]

· *F*(*T*) gravity $\longleftarrow$ **Extended teleparallel Lagrangian described by the torsion scalar T.**

[Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]

[Linder, Phys. Rev. D 81, 127301 (2010) [Erratum-ibid. D 82, 109902 (2010)]]

* "Teleparallelism" : One could use the Weitzenböck connection, which has no curvature but torsion, rather than the curvature defined by the Levi-Civita connection.

[Hayashi and Shirafuji, Phys. Rev. D 19, 3524 (1979) [Addendum-ibid. D 24, 3312 (1982)]]

- **Galileon gravity** ← $\Box\phi\left(\partial^{\mu}\phi\partial_{\mu}\phi\right)$

Longitudinal graviton (a branebending mode ϕ)

[Nicolis, Rattazzi and Trincherini, Phys. Rev. D 79, 064036 (2009)]

- **Massive gravity** ← **Graviton with a non-zero mass**

[de Rham and Gabadadze, Phys. Rev. D 82, 044020 (2010)]

[de Rham and Gabadadze and Tolley, Phys. Rev. Lett. 106, 231101 (2011)]

Review: [Hinterbichler, Rev. Mod. Phys. 84, 671 (2012)]